

An Analytical Solution for an Unsteady Boundary Layer Flow Along a Stretching Cylinder with Variable Thermal Conductivity

M.Y. Akl

*Engineering Mathematics and Physics Department
Faculty of Engineering, Benha University, Cairo, Egypt
E-mail: mhmdyhakl@gmail.com*

Abstract

The boundary layer unsteady flow along a continuously stretching cylinder immersed in a viscous and incompressible fluid is studied. A variable thermal conductivity is considered. The governing partial boundary layer equations in cylindrical form are first transformed into ordinary differential equations. These equations are solved analytically using the optimal modified Homotopy Asymptotic method in order to get a closed form solution for the dimensionless functions f , and θ .

Keywords: Optimal homotopy asymptotic method, stretching cylinder, boundary layer flow, unsteady flow.

1. Introduction

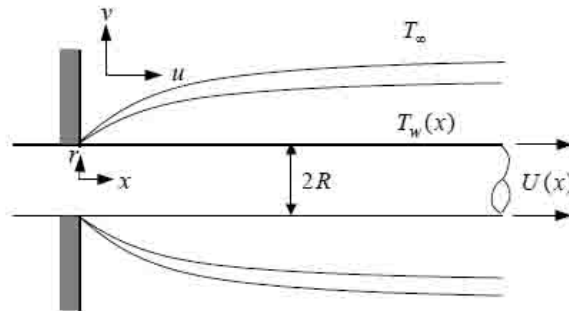
The boundary layer flow and heat transfer of stretching flat plates or cylinders are very important in fiber technology and extrusion processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. We have many applications such as the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning, drawing plastic films, and polymer extrusion. The quality of the final product depends on the rate of heat transfer at the stretching surface. Sakiadis [1] was the first to consider the boundary layer flow on a moving continuous solid surface. Crane [2] extended this concept to a stretching sheet with linearly varying surface speed and presented an exact analytical solution for the steady two-dimensional stretching of a surface in a quiescent fluid. Then many authors considered various aspects of this problem and obtained similarity solutions. A similarity solution is one in which the number of independent variables is reduced by at least one, usually by a coordinate transformation. The idea is analogous to dimensional analysis, but instead of parameters the coordinates ζ themselves are collapsed into dimensionless groups that scale the velocities (White, [15]). The boundary layer flow due to a stretching surface in a quiescent viscous and incompressible fluid when the buoyancy forces are taken into consideration have been considered by Daskalakis [8], Chen [11,12], Lin and Chen [10], Ali [13], Partha et al. [14], and Ishak et al. [5,8]. Lin and Shih [3,4], considered the boundary layer and heat transfer along horizontally and vertically moving cylinders with constant velocity and found that the similarity solutions could not be obtained due to the curvature effect of the cylinder. The case of stretching sheet is studied by Grubka and Bobba [5] and Ali [9], this work is extended by Ishak and Nazzar [23], to the case of

stretching cylinder . In this study we consider a stretching cylinder in an unsteady flow with variable thermal conductivity, and have been solved analytically.

2. Formulation of the Problem

Consider an unsteady, laminar, incompressible, and viscous flow on a continuous stretching cylinder as in figure (1).It is assumed that the stretching velocity $U(x) = (a x) \backslash (1-\gamma t)$, the surface temperature $T_w(x) = (b x) \backslash (1-\gamma t)$, where a , b , and γ are constants and the thermal conductivity $\alpha = \alpha_\infty (1 + \zeta \theta)$. The x -axis and y -axis are taken as shown in fig (1).The conservation equations for this case are:-

Figure 1: Physical Problem



$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = \quad (1)$$

$$\frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(ar \frac{\partial T}{\partial r} \right) \quad (3)$$

Subjected to The boundary conditions

$$u = U_w(x), \quad v = 0, T = T_w(x), \quad \text{at } r = R \quad (4)$$

$$u \rightarrow 0, T \rightarrow \infty \text{ as } r \rightarrow R$$

Where u and v are velocity components in the x and y directions, respectively, T is the fluid temperature and α is the thermal diffusivity. The continuity equation can be satisfied by introducing a stream function ψ , such that

$$u = \frac{1}{r} \frac{\partial \psi}{\partial r} \text{ and } v = \frac{1}{r} \frac{\partial \psi}{\partial x} . \text{ The momentum and energy equations can be transformed into the}$$

corresponding ordinary differential equations by the following transformations (Mahmoud and merkin [6], Ishak [20]) :-

$$\eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{U(x)}{\nu x}}, \psi = R \sqrt{U(x) \nu x} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

The transformed ordinary differential equations are:-

$$f''''(1 + 2\rho\eta) + 2\rho f'' + ff'' - f'^2 - \frac{\gamma}{\alpha} f' - \frac{\gamma}{\alpha} \frac{\eta}{2} f'' = 0 \quad (6)$$

$$\theta''(1 + 2\rho\eta) + 2\rho \theta'' + \zeta(1 + 2\rho\eta) (\theta''\theta + \theta'^2) + \zeta 2\rho \theta' \theta + p_r f \theta' - p_r f' \theta$$

$$-p_r \frac{\gamma}{a} \frac{\eta}{2} \theta' - p_r \frac{\gamma}{a} \theta = 0 \quad (7)$$

Where $(p_r) = (\nu \setminus \alpha_\infty)$ is the prandtl number

Subjected to the boundary conditions:-

$$f(0)=0, f'(0)=1, \theta(0)=1,$$

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0 \quad (8)$$

Where primes denotes differentiation with respect to η , and ρ denotes the curvature parameter defined as :-

$$\rho = \sqrt{\frac{\nu(1-\gamma t)}{aR^2}} \quad (9)$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined as :-

$$C_f = \frac{\tau_w}{\rho U^2 \setminus 2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad (10)$$

Where the surface shear stress τ_w and the surface heat flux q_w are given by :

$$\tau_w = \mu \left(\frac{\partial u}{\partial r} \right)_{r=R}, q_w = -k \left(\frac{\partial T}{\partial r} \right)_{r=R}, \quad (11)$$

With μ and k being the dynamic viscosity and the thermal conductivity, respectively .Using the similarity variables (5) we get :-

$$\frac{1}{2} C_f Re_x^{1/2} = f''(0), Nu_x \setminus Re_x^{1/2} = -\theta'(0), \quad (12)$$

Where $Re_x = Ux \setminus \nu$ is the local Reynolds number.

3. Optimal Homotopy Asymptotic Method(OHAM)

Consider a differential equation in the form:

$$L(u(t)) + N(u(t)) + g(t) = 0, B(u) = 0 \quad (13)$$

Where L is a linear operator, t denotes an independent variable, $u(t)$ is an unknown function, $g(t)$ is a known function, $N(u(t))$ is a nonlinear operator and B is a boundary operator. By means of OHAM a family of equations is constructed:

$$(1-p)[L(F(t,p)) + g(t)] - H(p)[L(F(t,p)) + g(t) + N(F(t,p))] = 0, B(F(t,p)) = 0 \quad (14)$$

where $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$ and $H(0)=0$, $F(t,p)$ is an unknown function. Obviously, when $p=0$, and $p=1$, we have::

$$F(t, 0) = u_0(t), F(t, 1) = u(t) \quad (15)$$

Then, as p increases from 0 to 1, the solution $F(t, p)$ varies from $u_0(t)$ to the solution $u(t)$, where $u_0(t)$ is obtained from (14) for $p=0$:

$$L(u_0(t)) + g(t) = 0, B(u_0) = 0 \quad (16)$$

The auxiliary function is chosen in the form;

$$H(p) = p C_1 + p^2 C_2 + \dots \quad (17)$$

Where C_1, C_2, \dots are constants which can be determined later.

Expanding $F(t,p)$ in a series with respect to p , we get:

$$F(t, p, C_i) = u_0(t) + \sum_{k \geq 1} u_k(t, C_i) p^k \quad (18)$$

Substituting (18) in (14), collecting the same powers of p , and equating each coefficient of p to zero, we obtain a set of differential equations with boundary conditions. Solving differential equations with boundary conditions

$u_0(t), u_1(t, C_1), u_2(t, C_2), \dots$ is obtained . Generally the solution of (13) can be determined in the form;

$$u^{(m)} = u_0(t) + \sum_{k=1}^m u_k(t, C_i) \tag{19}$$

Substituting (19) in (13) we get the following residual:

$$R(t, C_i) = L(u^{(m)}(t, C_i)) + g(t) + N(u^{(m)}(t, C_i)) \tag{20}$$

If $R(t, C_i) = 0$ then $u^{(m)}(t, C_i)$ is much closed to the exact solution to minimizing the occurred error for nonlinear problem, let;

$$(C_1, C_2, \dots, C_m) = \int_a^b R^2(t, C_1, C_2, \dots, C_m) dt \tag{21}$$

Where a and b are values depending on the given problem. The unknown constants C_i ($i = 1, 2, \dots, m$) can be determined from the conditions:

$$\frac{\partial J}{\partial c_1} = \frac{\partial J}{\partial c_2} = \dots = 0 \tag{22}$$

With these known constants, the approximate solution (of order m) (19) is well determined.

4. Solution using OHAM

Applying (14) into (8),(9) and (10) we get:

$$\begin{aligned} (1-p)[f'' + f'] - H_1(p)[f'''(1+2\rho\eta) + 2\rho f'' + f f'' - f'^2 - \gamma f' - \gamma \frac{\eta}{2} f'' - f'' - f'] &= 0 \\ (1-p)[\theta + \theta'] - H_2(p)[\theta''(1+2\rho\eta) + 2\rho\theta' + p r(f\theta' - f'\theta) + A(\theta + \theta') - \theta' - \theta] &= 0 \end{aligned} \tag{23}$$

Where primes denote differentiation with respect to η .

Since the first two equations in (23) are identical, then we take f, g, φ, H_1, H_2 and H_3 as following :

$$\begin{aligned} f &= f_0 + p f_1 + p^2 f_2, \\ \theta &= \theta_0 + p \theta_1 + p^2 \theta_2, \\ H_1(p) &= p C_1 + p^2 C_2, \\ H_2(p) &= p C_1 + p^2 C_2, \end{aligned} \tag{24}$$

Collecting same powers of p and solving the resulted set of differential equations we obtain;

$$\begin{aligned} f &= 1 - e^{-\eta} - \frac{1}{4} c l e^{-\eta} (-2A + 2Ae^{\eta} - 2A\eta + A\eta^2 + 4\eta^2 p) + \frac{1}{96} e^{-2\eta} (24 A c l^2 \\ &\quad + 48 A c l e^{\eta} - 240 A c l^2 e^{\eta} - 24 A^2 c l^2 e^{\eta} + 48 c 2 e^{\eta} A - 48 c l e^{2\eta} A \\ &\quad + 216 c l^2 e^{2\eta} A + 24 A^2 c l^2 e^{2\eta} - 48 c 2 e^{2\eta} A + 48 c l e^{\eta} A \eta \\ &\quad - 192 c l^2 e^{\eta} A \eta + 48 c 2 e^{\eta} A \eta - 24 c l^2 e^{\eta} A^2 \eta - 24 c l e^{\eta} A \eta^2 \\ &\quad + 48 c l^2 e^{\eta} A \eta^2 - 24 c 2 e^{\eta} A \eta^2 - 12 c l^2 e^{\eta} A^2 \eta^2 + 20 c l^2 e^{\eta} A^2 \eta^3 \\ &\quad - 3 c l^2 e^{\eta} A^2 \eta^4 + 96 c l^2 p - 384 c l^2 e^{\eta} p + 288 c l^2 e^{2\eta} p - 192 c l^2 \eta p \\ &\quad - 96 c l e^{\eta} \eta^2 p + 192 c l^2 e^{\eta} \eta^2 p - 96 c 2 e^{\eta} \eta^2 p - 96 c l^2 e^{\eta} A \eta^2 p \\ &\quad + 144 c l^2 e^{\eta} A \eta^3 p - 24 c l^2 e^{\eta} A \eta^4 p - 192 c l^2 e^{\eta} \eta^2 p^2 \\ &\quad + 256 c l^2 e^{\eta} \eta^3 p^2 - 48 c l^2 e^{\eta} \eta^4 p^2) \\ \theta &= e^{-\eta} + \frac{1}{4} c 3 e^{-2\eta} (4e^{\eta} \eta - 4e^{\eta} p r \eta + A e^{\eta} p r \eta^2 - 8 \zeta + 8 e^{\eta} \zeta - 8 e^{\eta} \eta p + \\ &\quad 4 e^{\eta} \eta^2 p - 8 \zeta p + 8 e^{\eta} \zeta p - 16 \eta \zeta p) + \frac{1}{96} e^{-3\eta} \end{aligned}$$

$$\begin{aligned}
& 96 c^3 e^{\eta} pr - 96 c^3 e^{2\eta} pr - 96 c^3 e^{\eta} pr^2 - 48 A \\
& c^3 e^{\eta} pr^2 + 96 c^3 e^{2\eta} pr^2 + 48 A c^3 e^{2\eta} pr^2 + 96 c^3 e^{2\eta} \eta - 288 c^3 e^{2\eta} \eta + \\
& 96 c^4 e^{2\eta} \eta - 48 A c^1 c^3 e^{\eta} pr \eta - 96 c^3 e^{2\eta} pr \eta - 96 A c^3 e^{2\eta} pr \eta + \\
& 48 A c^1 c^3 e^{2\eta} pr \eta + 384 c^3 e^{2\eta} pr \eta + 336 A c^3 e^{2\eta} pr \eta - pr \eta - 96 c^4 e^{2\eta} pr \eta - \\
& 96 A c^4 e^{2\eta} pr \eta + 48 A c^3 e^{\eta} pr^2 \eta - 96 c^3 e^{2\eta} pr^2 \eta - 96 A c^3 e^{2\eta} pr^2 \eta + \\
& 48 c^3 e^{2\eta} \eta^2 \\
& + 24 A c^3 e^{2\eta} pr \eta^2 - 96 c^3 e^{2\eta} pr \eta^2 - 192 A c^3 e^{2\eta} pr \eta^2 + 24 A c^4 e^{2\eta} pr \eta^2 + \\
& 48 c^3 e^{2\eta} pr^2 \eta^2 + 144 A c^3 e^{2\eta} pr^2 \eta^2 + 72 A^2 c^3 e^{2\eta} pr^2 \eta^2 + 24 A c^3 e^{2\eta} pr \eta^3 - \\
& 24 A c^3 e^{2\eta} pr^2 \eta^3 - 32 A^2 c^3 e^{2\eta} pr^2 \eta^3 + 3 A^2 c^3 e^{2\eta} pr^2 \eta^4 - 192 c^3 e^{\eta} \zeta + \\
& 960 c^3 e^{\eta} \zeta - 192 - c^4 e^{\eta} \zeta + 192 c^3 e^{2\eta} \zeta - 960 c^3 e^{\eta} \zeta + 192 c^4 e^{2\eta} \zeta + 96 c^3 pr \zeta - \\
& 384 c^3 e^{\eta} pr \zeta - 48 A c^3 e^{\eta} \zeta + 288 c^3 e^{2\eta} \zeta + 48 A c^3 e^{2\eta} \zeta - 384 \\
& c^3 e^{\eta} \eta \zeta + 192 c^3 e^{2\eta} \eta \zeta + 384 c^3 e^{\eta} pr \eta \zeta + 576 A c^3 e^{\eta} pr \eta \zeta - \\
& 192 c^3 e^{2\eta} pr \eta \zeta - 192 A c^3 e^{2\eta} pr \eta \zeta - 96 A c^3 e^{\eta} pr \eta^2 \zeta + 48 A c^3 e^{\eta} pr \eta^2 \zeta + \quad -96 \\
& 864 c^3 \zeta^2 - 786 c^3 e^{\eta} \zeta^2 \\
& c^3 e^{2\eta} \zeta^2 - 192 c^1 c^3 e^{\eta} pr \rho + 192 c^1 c^3 e^{2\eta} pr \rho - 192 c^3 e^{2\eta} \eta \rho + \\
& 960 c^3 e^{2\eta} \eta \rho - 192 c^4 e^{2\eta} \eta \rho - 192 c^1 c^3 e^{\eta} pr \eta \rho + 192 c^3 e^{2\eta} pr \eta \rho - \quad +96 \\
& 384 c^3 e^{2\eta} pr \eta \rho - 192 A c^3 e^{2\eta} pr \eta \rho \\
& c^3 e^{2\eta} \eta^2 \rho - 672 c^3 e^{2\eta} \rho + 96 c^4 e^{2\eta} \eta^2 \rho + 480 c^3 e^{2\eta} pr \eta^2 \rho + 528 A \\
& c^3 e^{2\eta} pr \eta^2 \rho + 96 c^3 e^{2\eta} \eta^3 \rho - 96 c^3 e^{2\eta} pr \eta^3 \rho - 240 A c^3 e^{2\eta} pr \eta^3 \rho + 24 A \\
& c^3 e^{2\eta} pr \eta^4 \rho - 192 c^3 e^{\eta} \rho \zeta + 960 c^3 e^{\eta} \rho \zeta - 192 c^4 e^{\eta} \rho \zeta + 192 c^3 e^{2\eta} \rho \zeta - 960 c^3 \\
& e^{2\eta} \rho \zeta + 192 c^4 e^{2\eta} \rho \zeta - 192 c^3 e^{\eta} pr \rho \zeta + 384 A c^3 e^{\eta} pr \rho \zeta + 192 c^3 e^{2\eta} pr \rho \zeta - 384 A c^3 \\
& e^{2\eta} pr \rho \zeta - 384 c^3 e^{\eta} \eta \rho \zeta + 3840 c^3 e^{\eta} \rho \zeta - 384 c^4 e^{\eta} \eta \rho \zeta - 192 c^3 e^{2\eta} pr \eta \rho \zeta + 192 c^3 \\
& pr \eta \rho \zeta - 384 c^3 e^{\eta} pr \eta \rho \zeta + 384 A c^3 e^{\eta} pr \eta \rho \zeta - 192 c^3 e^{2\eta} pr \eta \rho \zeta - 192 A c^3 \\
& e^{2\eta} pr \eta \rho \zeta - 1152 c^3 e^{\eta} \eta^2 \rho \zeta + 192 c^3 e^{2\eta} \eta^2 \rho \zeta + 768 c^3 e^{\eta} pr \eta^2 \rho \zeta + 1056 A c^3 \\
& e^{\eta} pr \eta^2 \rho \zeta + 48 A c^3 e^{2\eta} pr \eta^2 \rho \zeta - 192 A c^3 e^{\eta} pr \eta^3 \rho \zeta + 864 c^3 \zeta^2 \rho - 1536 c^3 e^{\eta} \zeta^2 \rho \\
& + 672 c^3 e^{\eta} \zeta^2 \rho + 3456 c^3 \eta \zeta^2 \rho - 1536 c^3 e^{\eta} \eta \zeta^2 \rho - 384 c^3 e^{2\eta} \eta \rho^2 + 960 c^3 e^{2\eta} \eta^2 \rho^2 - \\
& 448 c^3 e^{2\eta} \eta^3 \rho^2 + 48 c^3 e^{2\eta} \eta^4 \rho^2 + 2688 c^3 e^{\eta} \eta \zeta \rho^2 + 2304 c^3 e^{\eta} \eta \zeta \rho^2 - 384 \\
& c^3 e^{2\eta} \eta \zeta \rho^2 + 4224 c^3 e^{\eta} \eta^2 \zeta \rho^2 + 192 c^3 e^{2\eta} \eta^2 \zeta \rho^2 - 768 c^3 e^{\eta} \eta^3 \zeta \rho^2 + 672 c^3 \zeta^2 \rho^2 - 768 \\
& c^3 e^{\eta} \zeta^2 \rho^2 + 96 c^3 e^{2\eta} \zeta^2 \rho^2 + 1728 c^3 \eta \zeta^2 \rho^2 - 1563 c^3 e^{\eta} \eta \zeta^2 \rho^2 + 3456 c^3 \eta^2 \zeta^2 \rho^2
\end{aligned}$$

$$\text{Where: } A = \frac{\gamma}{a}$$

5. Results

Computations have been carried out for various values of the dynamic parameter (γ), the curvature parameter (ρ), the thermal conductivity (ζ) and the Prandtl number (pr).

Results for the skin friction $f'(0)$ are computed for various values of the dynamic parameter (γ) and the curvature parameter (ρ) in Table (1).

Table 1: Variation of $f''(0)$ for a various values of (ρ) at various values of (γ)

γ	$\rho = 0$	$\rho = 0.5$	$\rho = 1$
0.0	-1.00000	-1.24337	-1.37109
0.5	-1.17232	-1.38097	-1.4701
1.0	-1.32095	-1.49819	-1.55552

Results for the temperature surface gradient $-\theta'(0)$ are computed for various values of the dynamic parameters (γ) , (A) , Prandtl number (Pr) , the thermal conductivity (ζ) and the curvature parameter (ρ) in Tables (2) and (3).

Table 2: Variation of $-\theta'(0)$ for a various values of ρ, A, ζ and γ for $pr = 1$

pr	γ	ρ	ζ	$-\theta'(0)$
1	0	0	0.6	-0.92740
1	0	0	0.8	-0.86814
1	0	0	1	-0.59761
1	0	0.5	0.6	-1.13285
1	0	0.5	0.8	-1.01457
1	0	0.5	1	-0.96364
1	0	1	0.6	-1.90002
1	0	1	0.8	-1.4365
1	0	1	1	-1.24945
1	1	0	0.6	-1.22661
1	1	0	0.8	-1.0716
1	1	0	1	-0.839173
1	1	0.5	0.6	-1.45692
1	1	0.5	0.8	-1.19302
1	1	0.5	1	-1.03376
1	1	1	0.6	-2.25124
1	1	1	0.8	-1.70575
1	1	1	1	-1.07037

Table 3: Variation of $-\theta'(0)$ for a various values of ρ, A, ζ and γ for $pr = 2$

pr	γ	ρ	ζ	$-\theta'(0)$
2	0	0	0.6	-1.04767
2	0	0	0.8	-0.954924
2	0	0	1	-0.926166
2	0	0.5	0.6	-1.60857
2	0	0.5	0.8	-1.28896
2	0	0.5	1	-1.12618
2	0	1	0.6	-2.49076
2	0	1	0.8	-1.811
2	0	1	1	-1.70336
2	1	0	0.6	-1.29859
2	1	0	0.8	-1.12015
2	1	0	1	-0.971603
2	1	0.5	0.6	-1.87621
2	1	0.5	0.8	-1.43535
2	1	0.5	1	-1.2801
2	1	1	0.6	-2.6365
2	1	1	0.8	-2.08042
2	1	1	1	-0.885375

The variation of the transverse velocity (f) with the dimensionless variable (η) for different values of the dynamic parameter (γ) and the curvature parameter (ρ) is shown in figures (2-4). The variation of the horizontal velocity (f') with the dimensionless variable (η) for different values of the dynamic parameter (γ) and the curvature parameter (ρ) is shown in figures (5-7). The variation of temperature $\theta(\eta)$ with the dimensionless variable (η) for different values of the dynamic parameters (γ), (A), the prandtl number (Pr), the thermal conductivity (ζ) and the curvature parameter (ρ) is shown in figures (8-15).

Figure 2: Variation of the transverse velocity (f) for different values of $\rho=0, 0.5, 1$ at $\gamma=0, A=0$

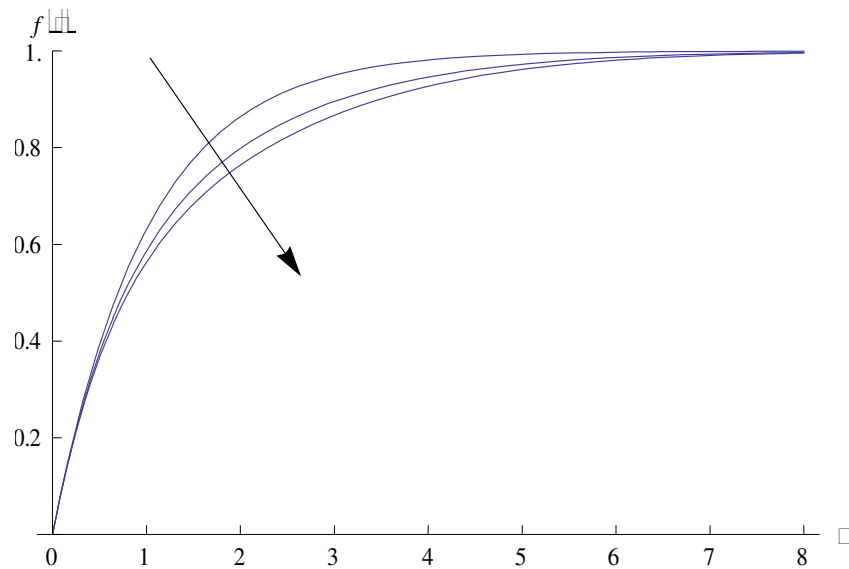


Figure 3: Variation of the transverse velocity (f) for different values of $\rho=0, 0.5, 1$ at $\gamma=0.5, A=$

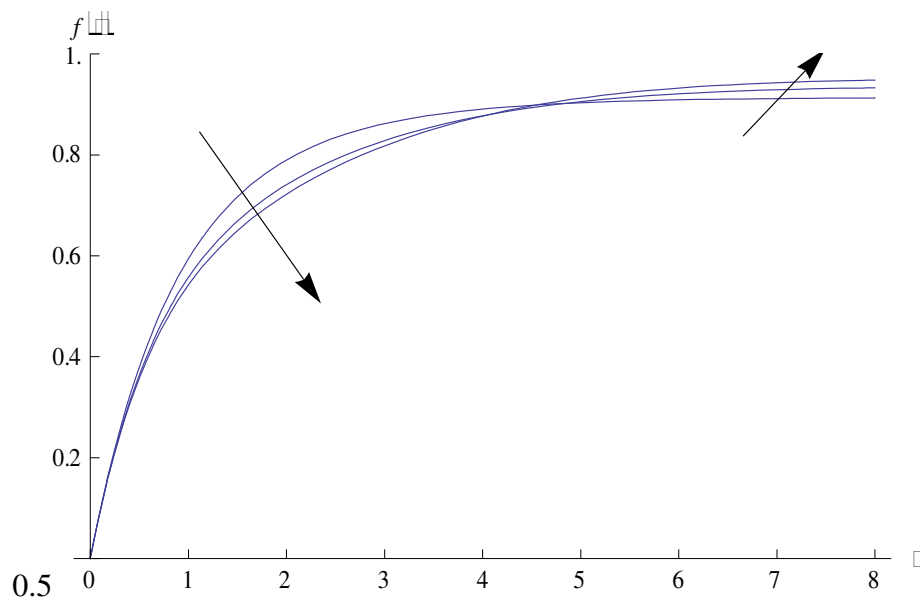


Figure 4: Variation of the transverse velocity (f) for different values of $\rho=0, 0.5, 1$ at $\gamma=1, A=1$

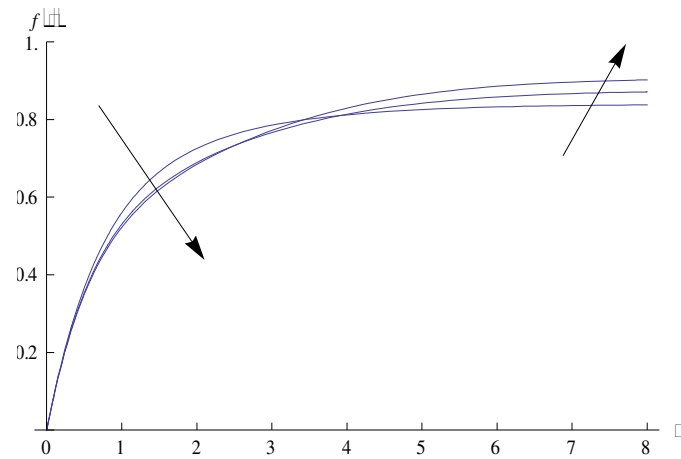


Figure 5: Variation of the horizontal velocity (f') for different values of $\rho=0, 0.5, 1$ at $\gamma=0, A=0$

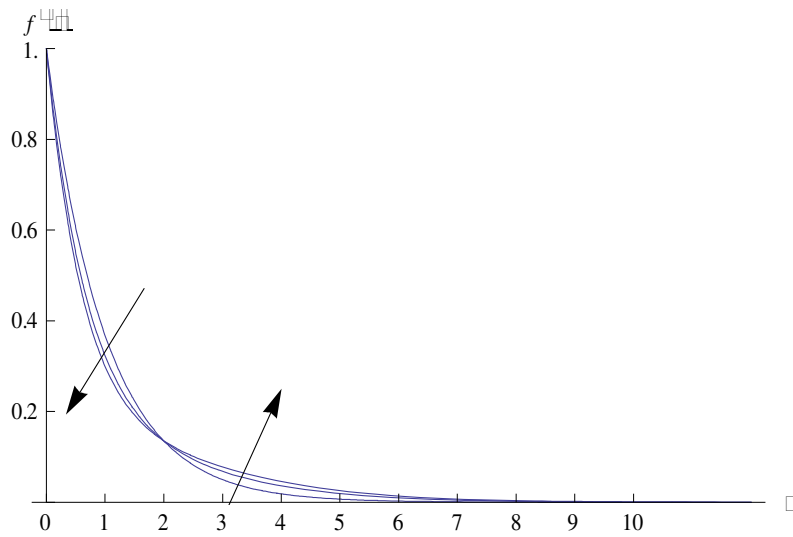


Figure 6: Variation of the horizontal velocity (f') for different values of $\rho=0, 0.5, 1$ at $\gamma=0.5, A=0.5$

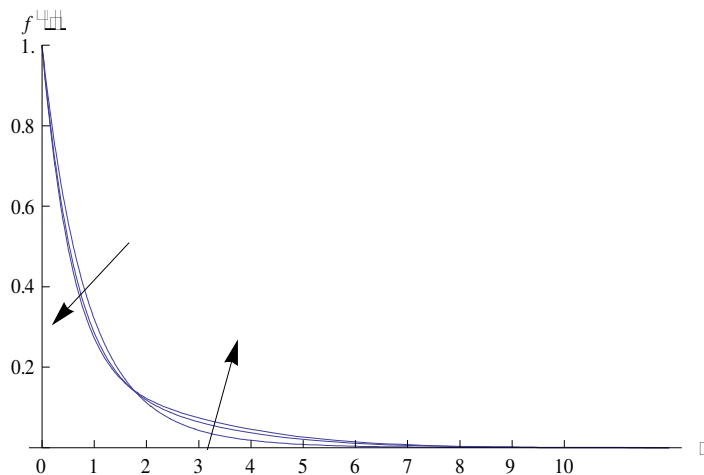


Figure 7: Variation of the horizontal velocity (f') for different values of $\rho = 0, 0.5, 1$ at $\gamma = 1, A = 1$

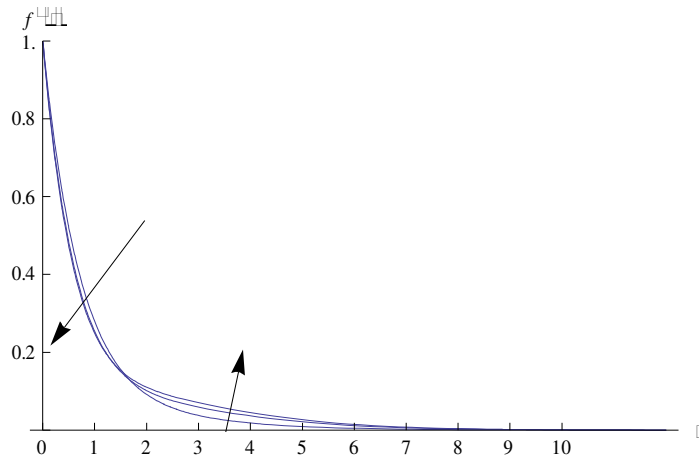


Figure 8: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 0, A = 0, \xi = 0, pr = 1$

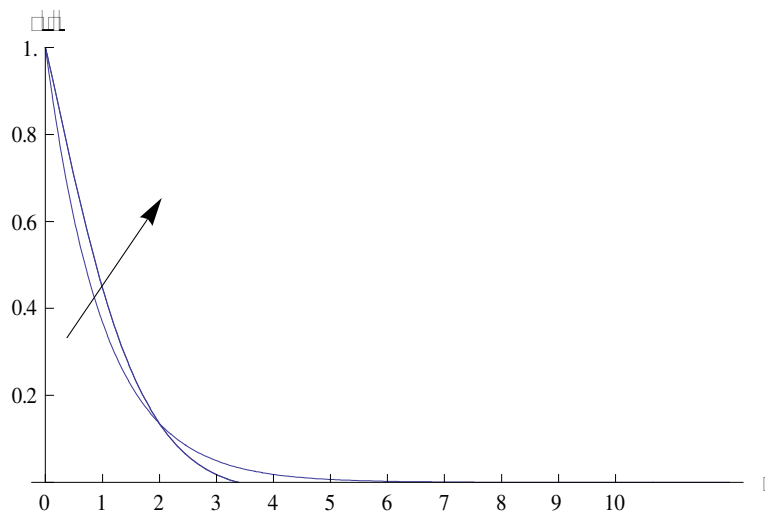


Figure 9: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 0, A = 0, \xi = 0.2, pr = 1$

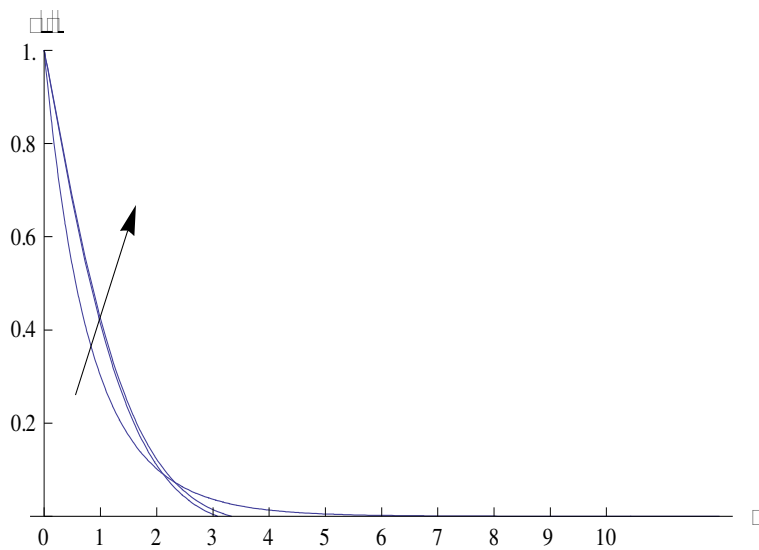


Figure 10: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 1, A = 1, \xi = 0, pr = 1$

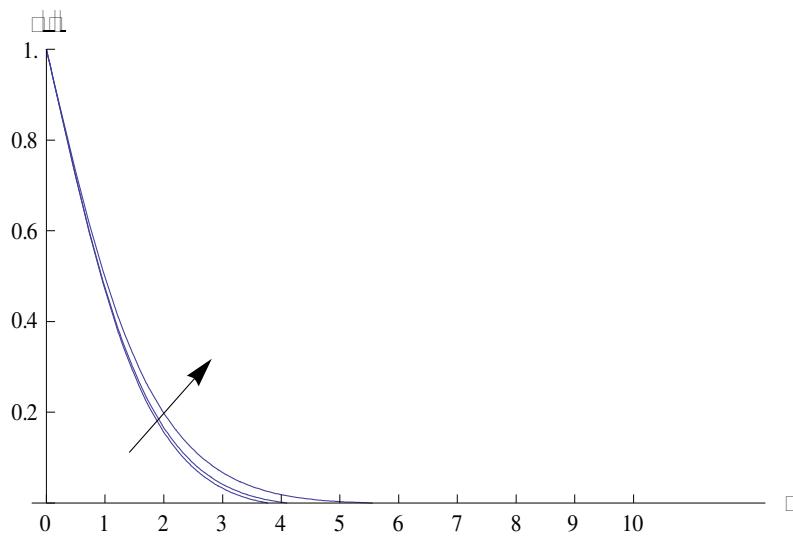


Figure 11: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 1, A = 1, \xi = 0.2, pr = 1$

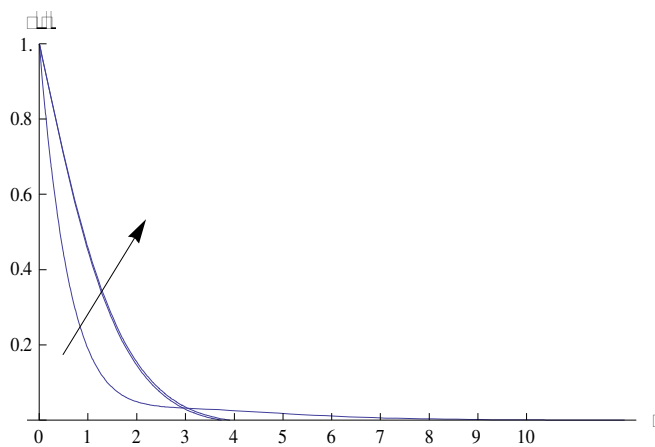


Figure 12: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 0, A = 0, \xi = 0.1, pr = 1$

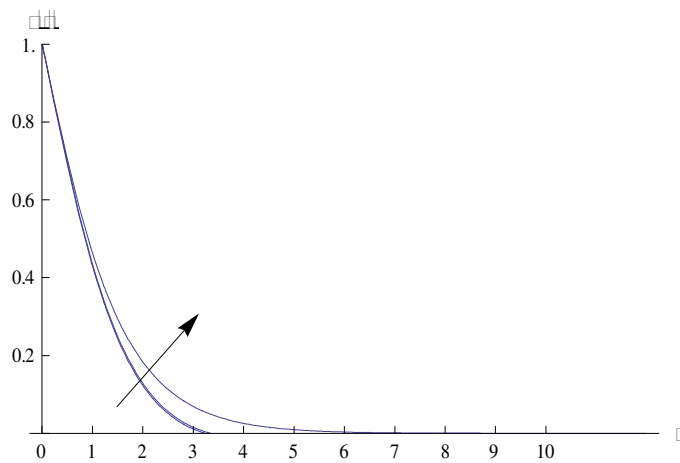


Figure 13: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 0, A = 0, \xi = 0.1, pr = 2$

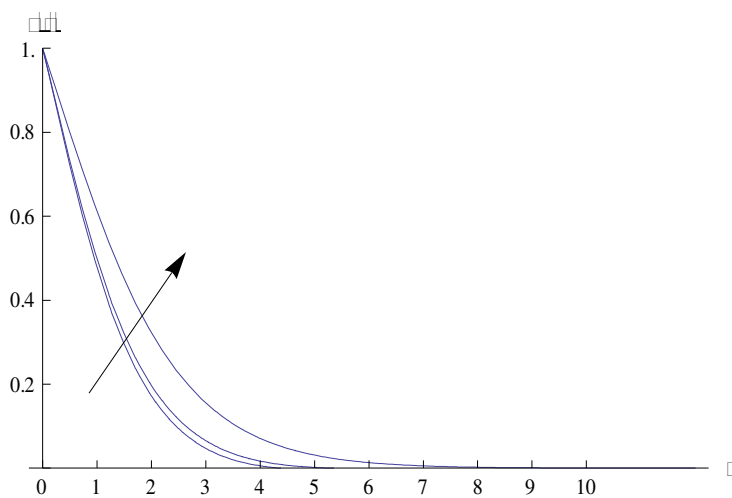


Figure 14: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 1, A = 1, \xi = 0.1, pr = 1$

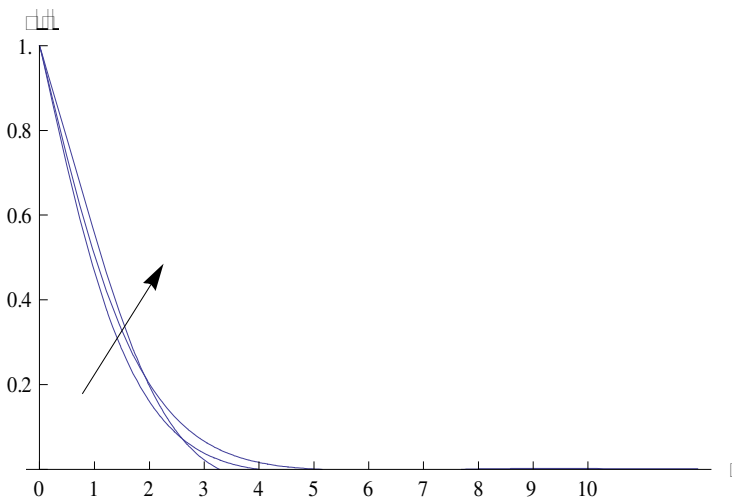
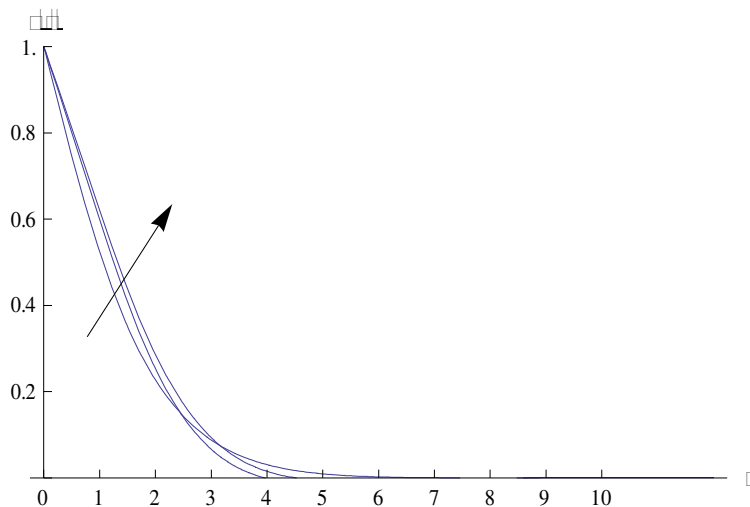


Figure 15: Variation of the temperature (θ) for different values of $\rho = 0, 0.5, 1$ at $\gamma = 1, A = 1, \xi = 0.1, pr = 2$



6. Discussions

The influence of the dynamic parameters (γ (A), the curvature parameter (ρ), the thermal conductivity (ζ) and the Prandtl number (Pr) on the dimensionless velocities (f), (f'), the skin friction coefficient $f''(0)$, the heat surface gradient $-\theta'(0)$ and the dimensionless heat (θ) are shown in tables (1,2) and figures (2-15).

Table (1) shows that the skin friction coefficient $f''(0)$ for all values of ρ and γ is negative which means the surface exerts a drag force on the fluid. Since equations (6) and (7) are uncoupled, then the Prandtl number does not affect on $f''(0)$. The absolute values of $f''(0)$ for all values of γ and ρ are greater than the values of $f''(0)$ when $\rho = 0$, which means the skin friction coefficient for the cylinder is greater than the plate. Also it is noticed that the skin friction $f''(0)$ increases as the curvature parameter (ρ) increases for all values of the dynamic parameter (γ) and then the skin friction decreases as the curvature parameter (ρ) increases for all values of the dynamic parameter (γ). Table(2, 3) show the heat transfer rate $-\theta'(0)$ increases as the curvature parameter (ρ) increases which means also the heat transfer rate at the surface for cylinder is greater than the heat transfer rate at the surface for the plate. The heat transfer rate $-\theta'(0)$ increases as the dynamic parameter (γ) increases. The heat transfer rate $-\theta'(0)$ decreases as the thermal conductivity (ζ) increases. Figures (2-4) show the transverse velocity profiles for various values of the curvature parameter (ρ), Figures (5-7) show the horizontal velocity profiles for various values of the curvature parameter (ρ). Figures (6-15) show the heat (θ) profiles for various values of the curvature parameter (ρ), the thermal conductivity (ζ) and the Prandtl number (Pr), it is clear that the heat increases as the curvature parameter increases for various values of the dynamic parameter (γ). Finally figures (2-15) show the satisfaction of initial boundary conditions which support the validity of the solution.

Conclusion

Optimal Homotopy Analysis Method has been applied to study the effects of the dynamic parameters (γ) and (A), the curvature parameter (ρ), the thermal conductivity (ζ) and the prandtl number (pr) on the velocity and the heat of the boundary layer unsteady flow. It is found:-

1. Closed form solutions for (f) and (θ) are obtained.
2. It is found that there are considerable effects for these parameters on the velocity and temperature.

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